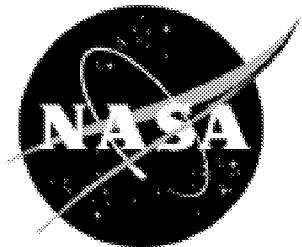


NASA/CR-2002-211635



# Design of a Dual Waveguide Normal Incidence Tube (DWNIT) Utilizing Energy and Modal Methods

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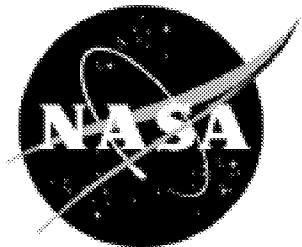
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## Abstract

This report presents an analysis on the partition design of the proposed Dual Waveguide Normal Incidence Tube (DWNIT). Some of the proposed usages of the DWNIT are:

1. Assessment of coupling relationships between resonators in close proximity
2. Evaluation of "smart liners"
3. Experimental validation for parallel element models
4. Investigation of effects of simulated angles of incidence through the introduction of higher order cross-sectional acoustic modes

To address this design challenge, an energy model was developed of the two chambers. This energy model was used to determine the SPL drop across the two chambers, through the use of an intensity transmission function for the chamber's partition. The model also allowed the chamber's lengthwise end samples to vary.

To model the partition, two intensity transmission function models were studied. The first intensity transfer function model used was a simple compressive model and the other was a pseudo-plate bending model.

The initial partition design (2" high, 16" long, 0.25" thick) was predicted to provide at least 160 dB SPL drop across the partition with the compressive model, and at least 240 dB SPL drop with the bending model. These results were produced when the structural damping loss in the bending model was 0.01 and the model end chamber sample transmissions were set to 0.1. This was done to analyze the partition under a close to worst-case scenario.

Since these results predicted more SPL drop than required, a plate thickness optimization algorithm was developed. The results of the algorithm routine indicated that a plate with the same height and length, but with a thickness of 0.1" and 0.05 structural damping loss, would provide an adequate SPL isolation between the chambers. This partition design would provide 210 dB and 125 dB SPL drop across the partition, utilizing the compressive and bending models, respectively. These results were also

computed with the end chamber sample transmissions set to 0.1. When more realistic end intensity transmission coefficients (e.g., 0.555) were used, the compressive and bending models predicted a partition isolation of 250 dB and 160 dB, respectively.

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**Figure 3-10.** SPL drop across a 0.1in (0.254cm) thick partition utilizing the Compressive Model with  $T_{1e}=T_{2e}=0.555$ .

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## List of Variables and Abbreviations

a	length of the of the partition plate
$a_e$	effective length of the of the partition plate
$A_n$	modal amplitude n
b	height of the partition plate
c	speed of sound in air
cm	centimeters
dB	decibel
dB_Drop	sound pressure level drop across the plate partition
D	beam bending stiffness
DWNIT	Dual Waveguide Normal Incidence Tube
E	plate modulus of elasticity
f	frequency
in	inches
Hz	hertz
I	sound intensity
$I_b$	moment of inertia of the plate
$I_{ref}$	reference impedance
$I_{12in}$	Sound intensity incident on surface 12
$I_{12ref}$	Sound intensity reflected from surface 12
$I_{12out}$	Sound intensity transmitted into surface 12
$I_{1ein}$	Sound intensity incident on surface 1e
$I_{1eref}$	Sound intensity reflected from surface 1e
$I_{1eout}$	Sound intensity transmitted into surface 1e
$I_{2ein}$	Sound intensity incident on surface 2e
$I_{2eref}$	Sound intensity reflected from surface 2e
$I_{2eout}$	Sound intensity transmitted into surface 2e
k	free acoustic wave number
$k_1$	wave number in the axial direction (x direction)
$k_2$	wave number in the normal direction (y direction)

$k_b$	bending plate wave number
$k_{be}$	effective bending plate wave number
kHz	kilohertz
$m$	mass per unit length of beam
Max	the Max function (finds the minimum of a given function)
Min	the Min function (finds the maximum of a given function)
NIT	Normal Incidence Tube
p.d.e.	partial differential equation
$P_{in}$	incident acoustic wave on the partition
$P_{out}$	transmitted acoustic wave from the partition
$q$	total number of bending modes used in model
$r_{air}$	characteristic impedance of air (equals $\rho c$ )
$r_{metal}$	characteristic impedance of metal used for the partition
R.H.S.	right hand side
RMS	root mean square
$S$	equals $S_{12}+S_{1e}$
$S_{12}$	partition's surface area
$S_{1e}$	surface area of surface 1e
$S_{2e}$	surface area of surface 2e
SIL	sound intensity level
$SIL_1$	sound intensity level in chamber 1 of the DWNIT
$SIL_2$	sound intensity level in chamber 2 of the DWNIT
SPL	sound pressure level
$\Delta SPL$	sound pressure level drop across partition plate
$t$	time
$T$	energy transmission coefficient or transfer function
$T_{12}$	Intensity transfer function for surface 12
$T_{1e}$	Intensity transfer function for surface 1e
$T_{2e}$	Intensity transfer function for surface 2e
$W$	acoustic power
$x$	axis along the length of the plate partition

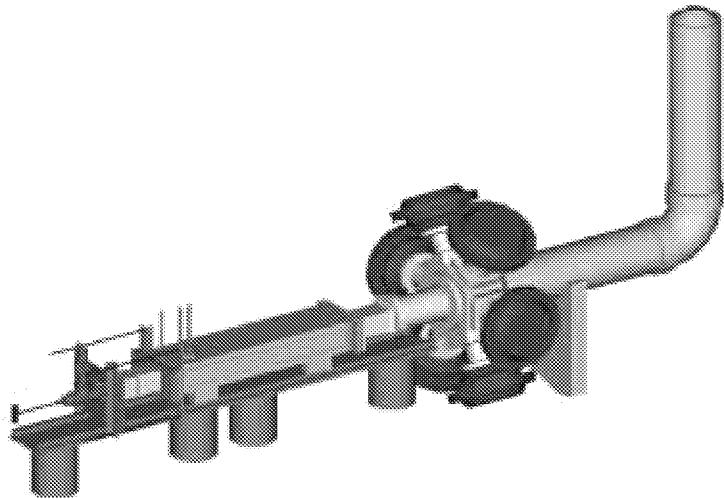
y	axis normal to the plate partition
Z	acoustic impedance
$\chi$	$\rho_c$ normalized nondimensional reactance
$\delta_m$	Kronica delta function (equals 1 when $r=n$ and zero when $r\neq n$ )
$\phi_n$	mode shape n
$\gamma$	structural damping loss factor
$\eta(x,t)$	plate partition displacement
$\theta$	$\rho_c$ normalized nondimensional resistance
$\rho$	density of air at room temperature
$\tau$	thickness of the plate partition
$\omega$	circular frequency (equals $2\pi f$ )
$\omega_n$	natural circular frequency
$\Psi_n$	equals $\int_0^a \phi_n(x) dx$

Throughout this report both English and SI units are used. English values and units are followed by the equivalent SI values and units in parentheses. The report follows an  $e^{+i\omega t}$  convention unless otherwise indicated. The symbol \* indicates a complex conjugate quantity. Quantities enclosed by | | indicate absolute values. Throughout this report, terms such as transfer function and transmission coefficient are used interchangeably, and indicate the ratio of output vs. input intensity in the frequency domain. References are located at the end of each chapter for the reader's convenience.

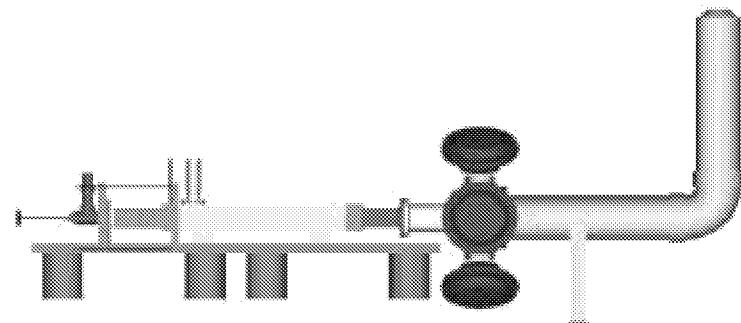
# 1. Introduction

## 1.1 Objective

The objective of this report is to analyze the partition design for the proposed NASA Langley Dual Waveguide Normal Incident Tube (DWNIT). The DWNIT allows simultaneous measurements of two end sample impedances. The proposed design adds a partition lengthwise to the current NASA Langley Normal Incident Tube (NIT).



**Figure 1-1.** Schematic of Current NIT.\* (perspective view)

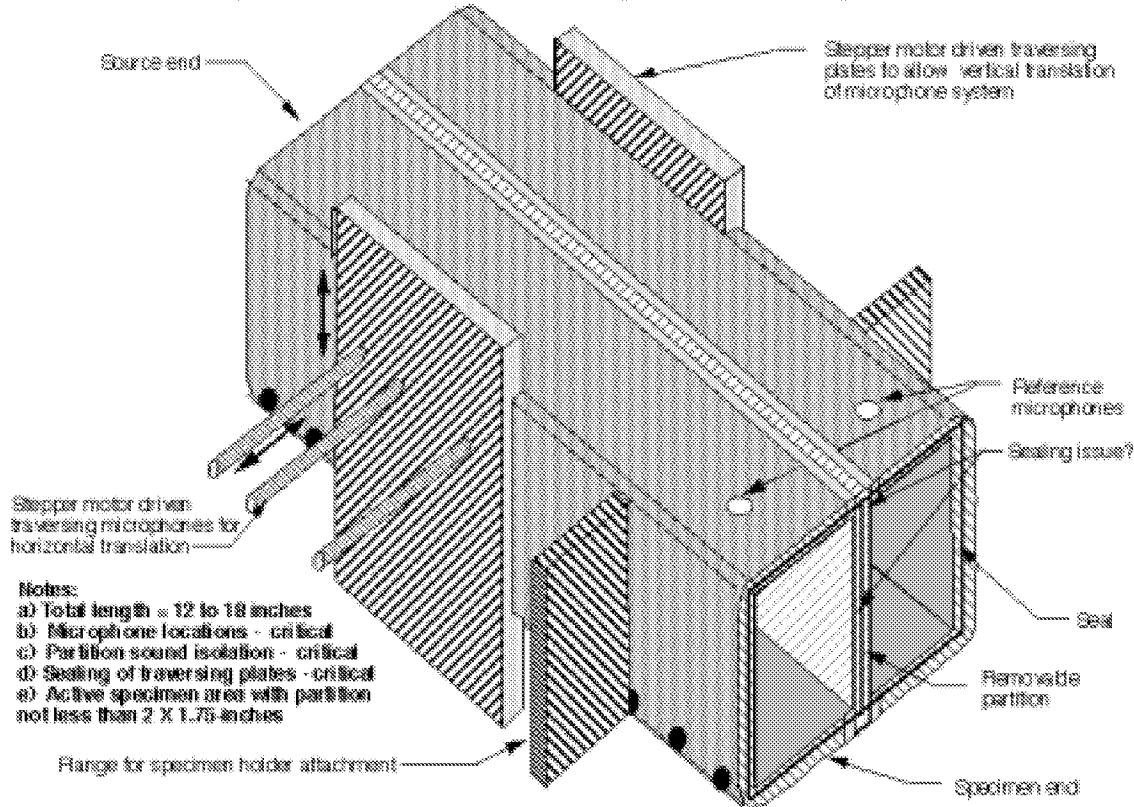


**Figure 1-2.** Schematic of Current NIT.\* (side view)

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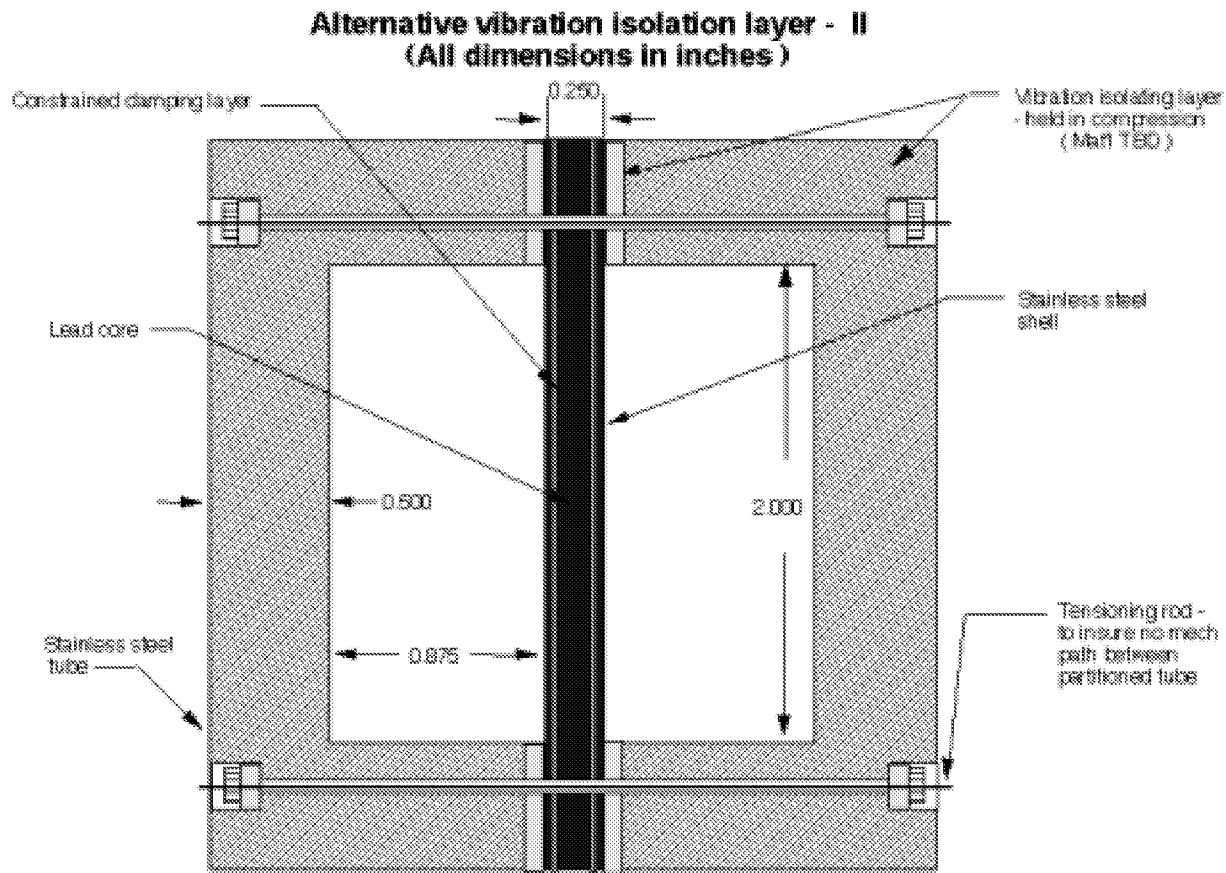
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Sketch of partitionable Z-tube with microphone traverse system



**Figure 1-3.** Schematic of Partition Design for the Proposed DWNIT (isometric view).<sup>†</sup>

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**Figure 1-4.** Schematic of Partition Design for the Proposed DWNIT (side view).<sup>†</sup>

Since the purpose of the DWNIT is to provide simultaneous and independent assessment of the impedances of two samples, the partition should be as thin as possible while providing at least 40 dB sound pressure level (SPL) isolation between each of its lengthwise chambers. Figures 1-1 and 1-2 show the current NASA Langley NIT and Figs. 1-3 and 1-4 show the proposed partition design for the DWNIT.

Some of the advantages, in the study of advanced liner concepts, provided by the DWNIT are:

1. Assessment of coupling relationships between resonators in close proximity
2. Evaluation of "smart liners"
3. Experimental validation for parallel element models
4. Investigation of effects of simulated angles of incidence through the introduction of higher order cross-sectional acoustic modes

## **1.2 Approach**

There are several design challenges associated with the DWNIT. This report will address one of the major challenges associated with the design of the DWNIT, specifically the design of the two-chamber partition. This partition must not allow acoustic energy from one chamber to leak into the other chamber and therefore distort the acoustic impedance of the end sample in the other chamber. The design objective is for the partition to provide at least 40 dB sound pressure level (SPL) drop isolation between the two chambers.

To address this design challenge, an energy model was developed of the two chambers. This energy model determined the SPL drop across the two chambers using an intensity transmission function for the chamber's partition. The model also allowed for the samples at the end of the two NIT chambers to vary.

To model the partition, two intensity transfer function models were studied. The first intensity transfer function model used was a simple compressive model.<sup>1</sup> This model assumes some of the incident acoustic wave is transmitted and the remaining portion is reflected. The impedance of the partition is simply associated with the interaction of the characteristic impedance ( $pc$ ) of air and that of the material associated with the partition. The solution of this model is provided by Kinsler et al in Ref. 1

The second intensity transfer function model was a structural acoustic interaction model.<sup>2</sup> This model assumed the partition would transmit sound from one chamber to the other through bending waves generated in the partition by the incident acoustic field. This model took into account the bending modes associated with a simply supported plate. The model also includes the radiation coincidence effect associated with the dispersion relationship.

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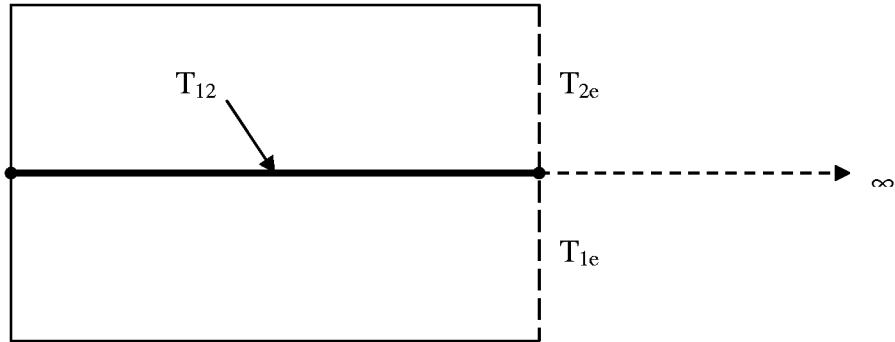
## **References**

1. Kinsler, L.E., Frey, A.R., Coppens, A.B., and Sanders, J.V., *Fundamental of Acoustics*, John Wiley & Sons, New York, 1982.
2. Fahy, F., *Sound Structural Vibration*, Academic Press, New York, 1998.

## 2. Theoretical Background

### 2.1 Energy Model Development

Figure 2-1 shows a schematic of the two chamber of the DWNIT. Variables  $T_{1e}$ ,  $T_{2e}$  and  $T_{12}$  are the intensity transmission functions of the end sample in chamber one, the end sample in chamber two, and the partition plate between chambers one and two, respectively.  $T_{1e}$  and  $T_{2e}$  are arbitrarily changed to study their variation effects on the SPL drop across the partition, while  $T_{12}$  is set by the transmission models developed in Section 2.2.



**Figure 2-1.** DWNIT model schematic.

To develop the energy model, intensity boundary conditions were applied to each of the three energy transmission surfaces. The intensity boundary conditions for surfaces  $T_{12}$ ,  $T_{2e}$ , and  $T_{1e}$  are given by:

$$\begin{aligned} I_{12out} &= I_{12in}T_{12} \\ I_{12ref} &= (1-T_{12})I_{12in} \end{aligned} \tag{2-1}$$

$$\begin{aligned}
I_{2eout} &= T_{2e} I_{2ein} \\
(I_{2ein} - I_{2eref})S_{2e} &= S_{12} I_{12out} \\
I_{2eref} &= (1 - T_{2e})I_{2ein}
\end{aligned} \tag{2-2}$$

$$\begin{aligned}
I_{1eout} &= I_{1ein} T_{1e} \\
I_{1eref} &= (1 - T_{1e})I_{1ein}
\end{aligned} \tag{2-3}$$

respectively.

The acoustic intensity (I) is related to the acoustic power (W) through

$$\begin{aligned}
I &= \frac{dW}{dS} \\
W &= \int I dS
\end{aligned} \tag{2-4}$$

where S is the total area. If the intensity is assumed to be constant through the surfaces, then  $W=IS$ . For this model it is assumed that there is an acoustic power in chamber one and no power source in chamber two. Therefore the total power input is

$$W = (I_{12in} - I_{12ref})S_{12} + (I_{1ein} - I_{1eref})S_{1e} \tag{2-5}$$

The total power  $I_{12in}$  and  $I_{1ein}$  can be related by:

$$\begin{aligned}
I_{12in} &= \frac{T_{12}}{T_{tot}} \frac{W}{S} \\
I_{1ein} &= \frac{T_{1e}}{T_{tot}} \frac{W}{S}
\end{aligned} \tag{2-6}$$

Therefore from Eq. (2-6)

$$\frac{I_{12}}{I_{1e}} = \frac{T_{12}}{T_{1e}} \tag{2-7}$$

For plane waves, the intensity is related to the RMS acoustic pressure ( $P_{rms}$ ) by

$$I = \frac{P_{rms}^2}{\rho c}$$

$$\therefore W = IS = \frac{P_{rms}^2}{\rho c} S \quad (2-8)$$

The SPL and Sound Intensity Level (SIL) are given by

$$SIL = 10 \log \left| \frac{I}{I_{ref.}} \right| \quad (2-9)$$

$$SPL = 10 \log \left| \frac{P_{rms}^2}{P_{ref.}^2} \right|$$

and for plane waves

$$SIL \approx SPL \quad (2-10)$$

For incoherent sources

$$P_{rms_{total}}^2 = P_{rms1}^2 + P_{rms2}^2 + \dots + P_{msn}^2 \quad (2-11)$$

To determine the SPL drop across the partition, the following procedure is followed. From Eqs. (2-1) through (2-3), the total intensity in chambers one ( $I_1$ ) and two ( $I_2$ ) can be determined. The SIL for each chamber can be used instead of the SPL, and subtracting  $SIL_1$  from  $SIL_2$  will give the SPL drop across the partition.

Therefore,  $I_1$  is given by

$$I_1 = I_{12in} + I_{12ref} + I_{1ein} + I_{1eref} \quad (2-12)$$

Using Eqs. (2-1), (2-3), (2-7), and (2-12) produces

$$I_1 = \left| \left( 2 - T_{12} \right) + \left( 2 - T_{1e} \right) \right| \frac{T_{1e}}{T_{12}} I_{12in} \quad (2-13)$$

$I_2$  is given by

$$I_2 = I_{2ein} + I_{2eref} \quad (2-14)$$

using Eqs. (2-2) and (2-14) produces

$$I_2 = \left( 2 - T_{2e} \right) \frac{S_{12}}{S_{2e}} \frac{T_{12}}{T_{2e}} I_{12in} \quad (2-15)$$

The SPL drop across the partition is

$$\Delta SPL \approx SIL_1 - SIL_2 = 10 \log \left| \frac{I_1}{I_{ref}} \right| - 10 \log \left| \frac{I_2}{I_{ref}} \right| = 10 \log \left| \frac{I_1}{I_2} \right| \quad (2-16)$$

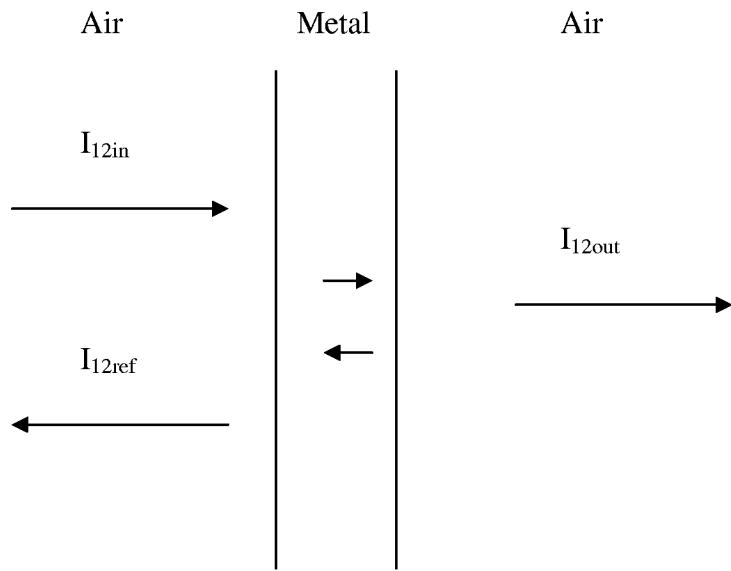
Therefore  $\Delta SPL$  is

$$\Delta SPL \approx 10 \log \left| \frac{\left| \left( 2 - T_{12} \right) + \left( 2 - T_{1e} \right) \right| \frac{T_{1e}}{T_{12}}}{\left| \left( 2 - T_{2e} \right) \frac{S_{12}}{S_{2e}} \frac{T_{12}}{T_{2e}} \right|} \right| \quad (2-17)$$

## 2.2 Chamber Coupling Intensity Transfer Function

### 2.2.1 Compressive Model

The energy model developed in Section 2.1 required as an input the intensity transmission function of the partition ( $T_{12}$ ). The simplest model for this transfer function is a compressive acoustic waves incident on an infinitely long metal as shown in Fig. 2-2.



**Figure 2-2. Compressive model schematic.**

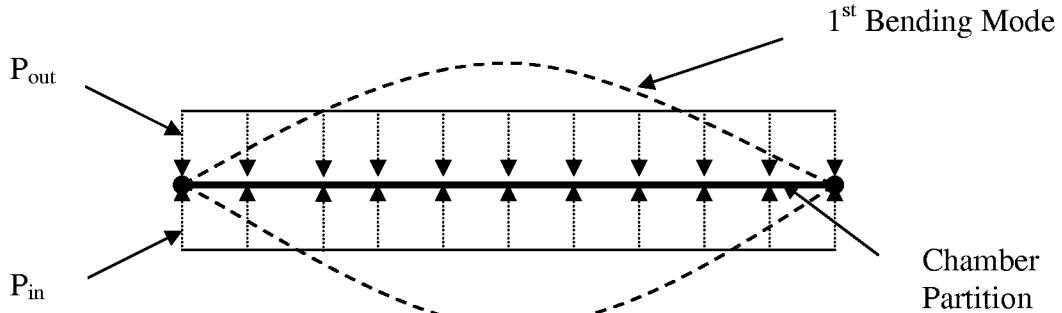
The solution for the transmission function  $T_{12}$  is given by Kinsler et al<sup>1</sup> as

$$T_{12} = \left| \frac{2r_{air}}{r_{metal} \sin(k_{metal} \tau)} \right| \quad (2-18)$$

where  $r_{air}$  and  $r_{metal}$  are the characteristic impedance (pc) of air and the metal used, respectively, and  $k_{metal}$  and  $\tau$  are the wave number and thickness of the partition metal, respectively.

## 2.2.2 Pseudo-Plate Model (Bending Model)

In order to better assess the energy transmitted from one chamber of the DWNIT to the other, a model that accounts for the structural bending response of the partition was needed. A simply supported pseudo-plate bending model for the chamber's partition is presented in this section. Figure 2-3 shows a schematic of the model used in this section.



**Figure 2-3.** Bending model schematic.

The sound structure interaction under various conditions has been studied by a number of authors. The analysis that follows uses some of the analysis elements of Fahy.<sup>2</sup> The governing differential equation of a beam is given by:

$$D \frac{\partial^4 \eta(x, t)}{\partial x^4} + m \frac{\partial^2 \eta(x, t)}{\partial t^2} = (P_{in} - P_{out}) \rho e^{i\omega t} \quad (2-19)$$

where  $D$  and  $m$  are the dynamic stiffness and mass per unit length, respectively, of the beam. For a beam,  $D$  is given by  $EI_b(1+i\gamma)$ , where  $E$  is the modulus of elasticity,  $I_b$  is the moment of inertia, and  $\gamma$  is the structural damping loss factor.  $P_{in}$  and  $P_{out}$  are the incident and transmitted acoustic pressures, respectively. The solution of Eq. (2-19) can be written as a summation of modal amplitudes multiplied by their mode shapes, which takes the following form:

$$\eta(x, t) = \sum_{n=1}^q A_n(\omega) \phi_n(x) e^{i\omega t} = A_n \phi_n e^{i\omega t} \quad (2-20)$$

where the summation symbol is eliminated for simplicity and the Einstein's summation convention is used, where repeated indices imply summation.  $\phi_n$  is the *in vacuo* mode shape for a given set of boundary conditions. The mode shapes can be determined by setting the R.H.S of Eq. (2-19) to zero (*in vacuo* or homogeneous p.d.e), and solving the p.d.e for a given set of boundary conditions. The natural frequency is calculated by inserting the mode shape solution back into the homogeneous p.d.e and solving for the natural frequency  $\omega_n$ . For a simply supported beam, the boundary conditions at  $x = 0$  and  $x = a$  are

$$\begin{aligned}\eta(0, t) &= \eta(a, t) = 0 \\ M_z(0, t) &= \frac{\partial^2 \eta(0, t)}{\partial x^2} = M_z(a, t) = \frac{\partial^2 \eta(a, t)}{\partial x^2} = 0\end{aligned}\quad (2-21)$$

and the mode shape and natural frequency are

$$\begin{aligned}\phi_n(x) &= \sin\left(\frac{n\pi}{a}x\right) = \sin(k_b x) \\ \omega_n &= \left(\frac{n\pi}{a}\right)^2 \sqrt{\frac{D}{m}} = (k_b)^2 \sqrt{\frac{D}{m}} \\ k_b &= \frac{n\pi}{a}\end{aligned}\quad (2-22)$$

If the beam natural frequencies in Eq. (2-22) were used to model the plate partition, the model would make the plate too flexible. Therefore, a pseudo-plate model is approximated by matching the simply supported plate natural frequencies to that of a simply supported beam. The natural frequencies for a simply supported plate are given by

$$\omega_{nq} = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{q\pi}{b}\right)^2 \sqrt{\frac{D}{m}} \quad (2-23)$$

Setting the natural frequency from Eq. (2-22) equal to that of Eq. (2-23) yields

$$\left| \frac{n\pi}{a_e} \right|^2 = \left| \frac{n\pi}{a} \right|^2 + \left| \frac{q\pi}{b} \right|^2 \quad (2-24)$$

where  $a$  in Eq. (2-22) has been replaced by an effective length  $a_e$ . Setting  $q$  in Eq. (2-24) to unity and solving for  $a_e$  yields

$$a_e = \frac{abn}{\sqrt{a^2 + b^2 n^2}} \quad (2-25)$$

Therefore the natural frequencies and mode shapes used in this pseudo-plate model are

$$\omega_n = \left| \frac{n\pi}{a_e} \right|^2 \sqrt{\frac{D}{m}} = (k_{be})^2 \sqrt{\frac{D}{m}}$$

and

$$\phi_n(x) = \sin \left| \frac{n\pi}{a_e} x \right| = \sin(k_{be} x)$$
(2-26)

where  $k_{be}$  is now

$$k_{be} = \frac{n\pi}{a_e} \quad (2-27)$$

Inserting Eq. (2-20) into Eq. (2-19), then multiplying by  $\phi_r$  and integrating over the domain from  $x=0$  to  $x=a$  produces

$$\int_0^a \phi_r D A_n \frac{\partial^4 \phi_n}{\partial x^4} dx - \int_0^a \phi_r \omega^2 m A_n \phi_n \phi_n dx = \int_0^a \phi_r (P_{in} - P_{out}) b dx \quad (2-28)$$

This equation is called the weak modal formulation of the p.d.e. The modal orthogonality conditions are given by

$$\begin{aligned} \int_0^a m \phi_r \phi_n dx &= \delta_{rn} \\ \int_0^a D \phi_r \frac{\partial^4 \phi_n}{\partial x^4} dx &= \delta_{rn} \omega_r \end{aligned} \quad (2-29)$$

where  $\delta_{rn}$  is the Kronecker Delta function, which has a value of one when r and n are equal, and zero when they are not equal. Applying the conditions in Eq. (2-29) into Eq. Eq. (2-28) produces

$$A_n \omega_n^2 - \omega^2 A_n = (P_{in} - P_{out}) b \int_0^a \phi_n(x) dx = (P_{in} - P_{out}) b \psi_n \quad (2-30)$$

where  $\psi_n = \int_0^a \phi_n(x) dx$ , and repeated indices in this equation do not indicate summation.

$\psi_n$  has a value of one when n is odd and zero when n is even. Therefore, only symmetric modes transmit acoustic energy from one chamber of the DWNIT to the other. Solving for  $A_n$  in Eq. (2-30) yields

$$A_n = \frac{(P_{in} - P_{out}) b \psi_n}{\omega_n^2 - \omega^2} \quad (2-31)$$

From Eqs. (2-31) and (2-20), the dynamic displacement response of the plate is given by

$$\eta(x, t) = (P_{in} - P_{out}) \sum_{n=1}^m \frac{b \psi_n \phi_n(x)}{\omega_n^2 - \omega^2} e^{i \omega t} \quad (2-32)$$

The plate velocity response is given as

$$\dot{\eta} = \frac{\partial \eta}{\partial t} = i \omega (P_{in} - P_{out}) \sum_{n=1}^m \frac{b \psi_n \phi_n(x)}{\omega_n^2 - \omega^2} e^{i \omega t} \quad (2-33)$$

To determine the radiated field, the maximum velocity function in the pseudo-plate was calculated using the no-penetration boundary conditions. Consequently, Eq. (2-33) becomes

$$\dot{\eta} = \alpha(P_{in} - P_{out})e^{i\omega t} \quad (2-34)$$

where

$$\alpha = \text{Max}_{n=1}^{\infty} i\omega \frac{\psi_n \phi_n(x)}{\omega_n^2 - \omega^2} \quad (2-35)$$

where the Max function returns the complex number with the largest modulus (magnitude). The acoustic pressure field generated by the vibrating plate is given by

$$p_{out} = P_{out} e^{i(\omega t - k_1 x - k_2 y)} \quad (2-36)$$

The no-penetration boundary condition requires that the acoustic particle velocity and the plate velocity must match. Therefore, applying this boundary condition and using Eq. (2-36) yields

$$\dot{\eta}(x,0,t) = -\frac{1}{i\omega\rho} \frac{\partial p_{out}}{\partial y} = \frac{k_2}{\omega\rho} p_{out}(x,0,t) \quad (2-37)$$

$k_2$  must satisfy the radiation dispersion relationship

$$k^2 = k_1^2 + k_2^2 = \left| \frac{\omega}{c} \right|^2$$

or

$$k_2 = \sqrt{\left| \frac{\omega}{c} \right|^2 - \omega^2} \frac{m}{D}^{\frac{1}{2}}$$
(2-38)

where

$$k_1 = k_b = \sqrt{\omega \left( \frac{m}{D} \right)^{\frac{1}{2}}} \quad (2-39)$$

Using Eqs. (2-34), (2-37), (2-38) and rearranging yields

$$\frac{P_{out}}{P_{in}} = \frac{\alpha}{\left| \alpha + \frac{k_2}{\omega \rho} \right|} \quad (2-40)$$

The intensity transmission coefficient is given by

$$T_{12} = \frac{I_{12out}}{I_{12in}} = \frac{P_{12out} u_{12out}^*}{P_{12in} u_{12in}^*} = \frac{P_{12out}^* \frac{P_{12out}^*}{Z^*}}{P_{12in}^* \frac{P_{12in}^*}{Z^*}} = \left| \frac{P_{12out}}{P_{12in}} \right|^2 \quad (2-41)$$

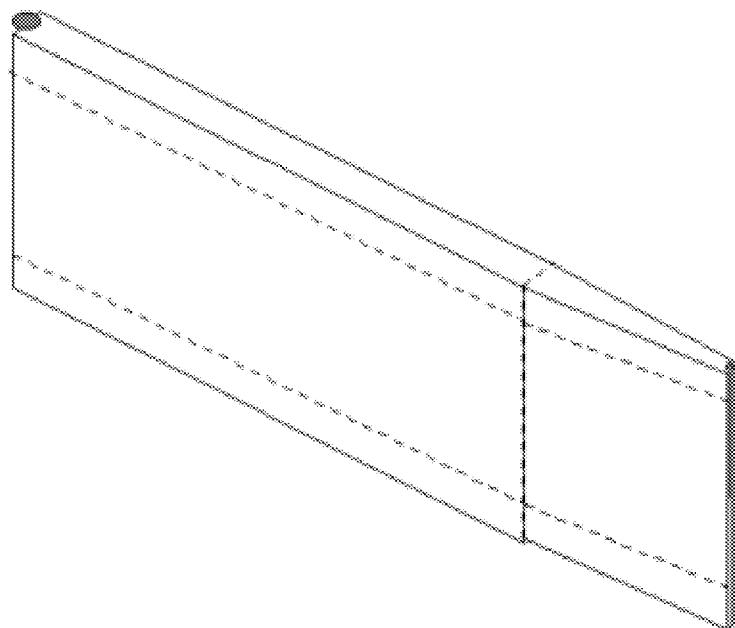
Therefore using Eqs. (2-40) and (2-41), the intensity transmission coefficient is

$$T_{12} = \left| \frac{\alpha}{\left| \alpha + \frac{k_2}{\omega \rho} \right|} \right|^2 \quad (2-42)$$

where  $k_2$  must be greater than zero for acoustic energy to be transmitted through the partition.

## 2.3 Partition Thickness and Structural Damping Loss Factor

Two important factors that must be considered in the design of the chamber partition, are the amount of structural damping loss factor to be added to the partition and the thickness of the partition plate. Near the sample the partition is tapered to a point (as seen in Fig. 2-4, in order to minimize the effect of the partition pressing on the sample. Intuitively, the thicker the partition is made the greater the SPL drop across the plate and the slope of the taper required. If the slope of the taper is too large, the acoustic field will be disturbed and the educed acoustic impedance will be the coupling effect between the near field effects of the partition taper and the end sample tested. Therefore, the partition thickness should be designed as thin as possible, while still providing an adequate SPL drop across the partition.



**Figure 2-4.** End taper design of plate partition.\*

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Intuitively, it can also be seen that by increasing the damping loss factor, the SPL drop will increase, since a greater amount of the total incident energy will be dissipated rather than transmitted through the plate partition. Consequently, the partition could be made thinner by simply adding more damping. Nevertheless, this effect is also undesirable because it decreases the amount of the reflected energy back into the incident energy chamber. This in effect would “soften” the wall, shifting the crossectional modes from the desired “hard” wall conditions. Moreover, adding significant damping will increase the required acoustic power from the drivers to achieve the equivalent SPLs desired in the chambers. Therefore, the amount of damping should also be minimized.

This problem lends itself to a constrained minimization subject to a set of SPL drop, damping, and plate thickness constraints.<sup>3</sup> The goal is to determine the minimum SPL drop (must not be less than 40 dB) that can be achieved under the constraints of

$$\begin{aligned} 0.05 \leq \tau \leq 0.35 \\ 0.01 \leq \gamma \leq 0.1 \end{aligned} \quad (2-43)$$

where the thickness ( $\tau$ ) is given in inches in Eq. (2-43). The loss factor ( $\gamma$ ) can be changed by adding commercially available viscoelastic damping polymers.<sup>4</sup> A MATLAB program was written that searches for the minimum SPL drop in the  $\Delta$ SPL power spectrum utilizing the models developed in this chapter for the partition.

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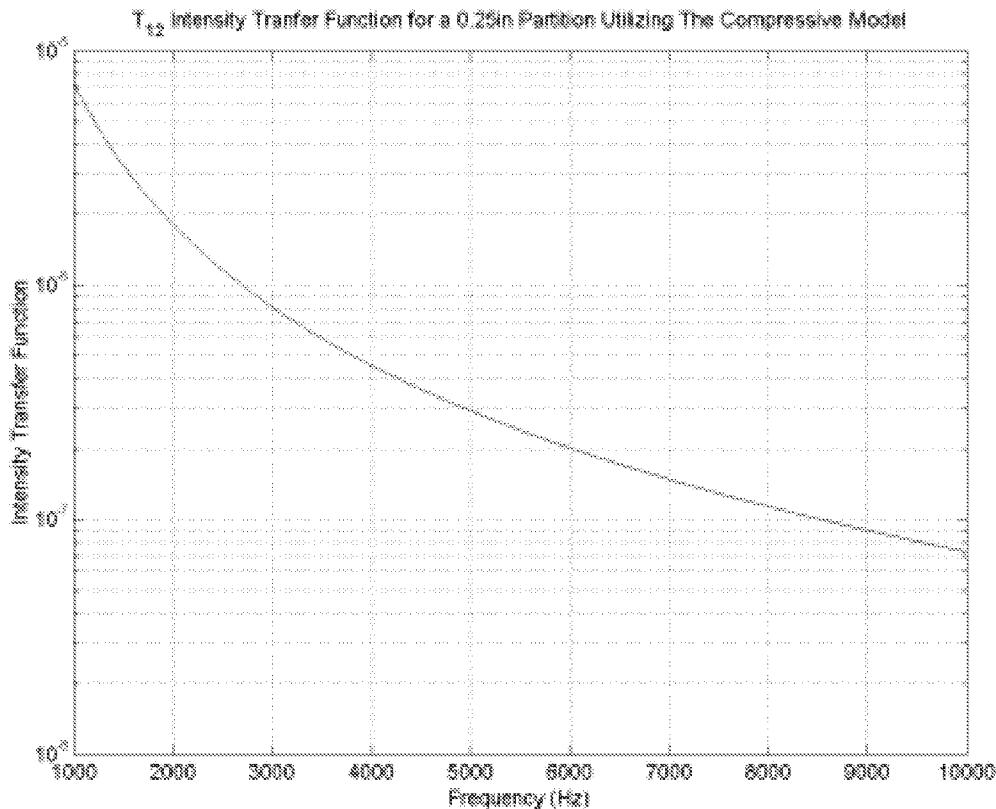
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4. Viscoelastic Damping Polymers, 3M Bonding Systems Division, St. Paul, 1999.

### 3. Results

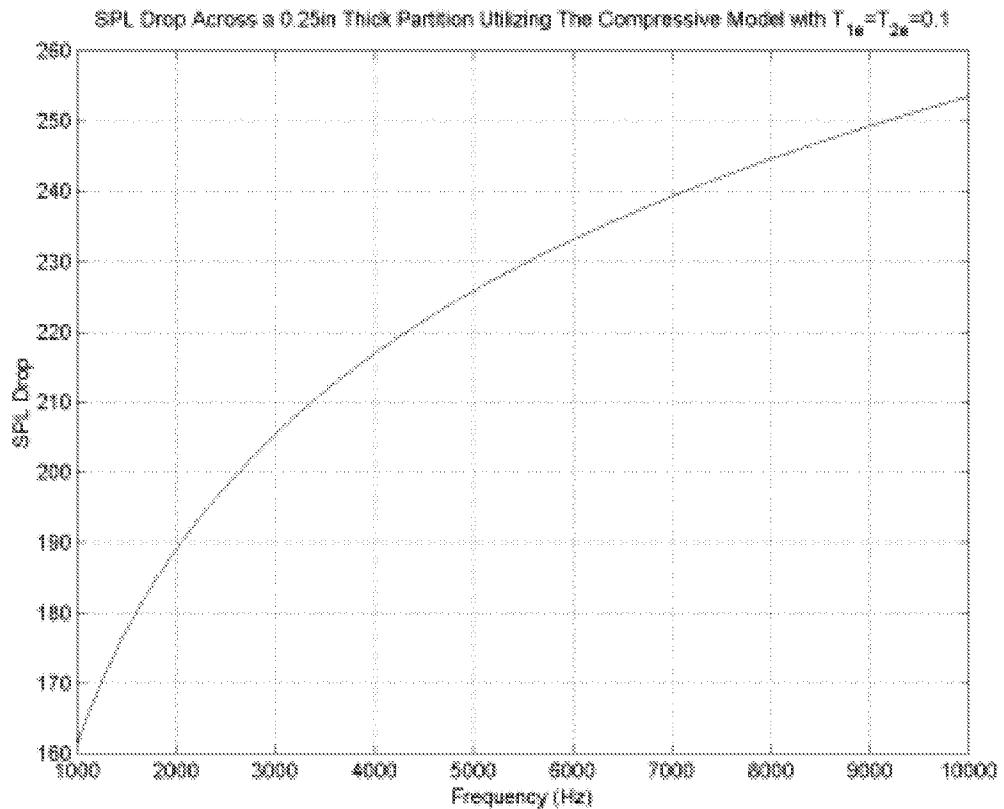
The initial design of the partition plate was 2 in (5.08cm) high by 16 in (40.64cm) long and 0.25 in (0.635cm) thick. The chosen plate material was steel and the frequency range of interest was 1 to 10 kHz. All the intensity transfer functions can assume values between zero and one. From Chapter 2, an intensity transfer function value of zero indicates that all the incident acoustic energy is reflected, while a value of one indicates that all the acoustic energy is transmitted.

The partition design requirement was to achieve at least 40dB SPL drop (isolation) between the two chambers of the DWNIT. From the energy model developed in Section 2.1, it can intuitively be seen from Eq. (2-17) that when  $T_{1e}$  and  $T_{2e}$  are small, the SPL drop across the partition is at its minimum. Therefore, these conditions would be close to the worst-case scenario the partition will experience.



**Figure 3-1.** Intensity Transfer Function for a 0.25in (0.635cm) thick partition utilizing the Compressive Model.

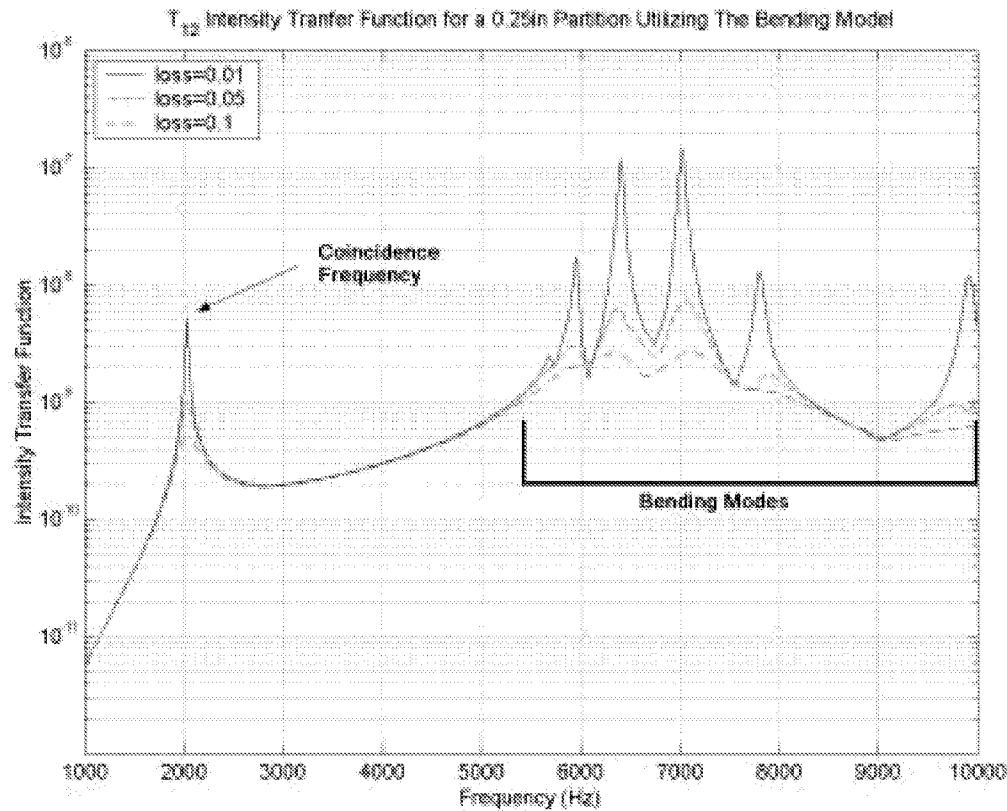
Figure 3-1 shows the transmission function  $T_{12}$  for a 0.25in (0.635cm) thick steel partition utilizing the compressive model. The figure shows a small magnitude for the transmission, indicating a small transfer of energy between the two chambers of the DWNIT. The figure also shows that the transmission decreases with increasing frequency. This well known effect occurs, because as the frequency of the acoustic wave increases, the wavelength decreases. The shorter wavelengths are better able to “see” the plate thickness and therefore a greater percentage of the acoustic energy gets reflected.



**Figure 3-2.** SPL drop across a 0.25in (0.635cm) thick partition utilizing the Compressive Model with  $T_{1e}=T_{2e}=0.1$ .

Figure 3-2 shows the SPL drop across a 0.25in (0.635cm) thick steel partition utilizing the compressive model with  $T_{1e}=T_{2e}=0.1$ . The figure shows that the minimum SPL drop across the partition is 160dB, well above the 40dB minimum design requirement. The figure shows the SPL drop increasing with frequency, which is due to the partition’s intensity transfer function decrease with frequency.

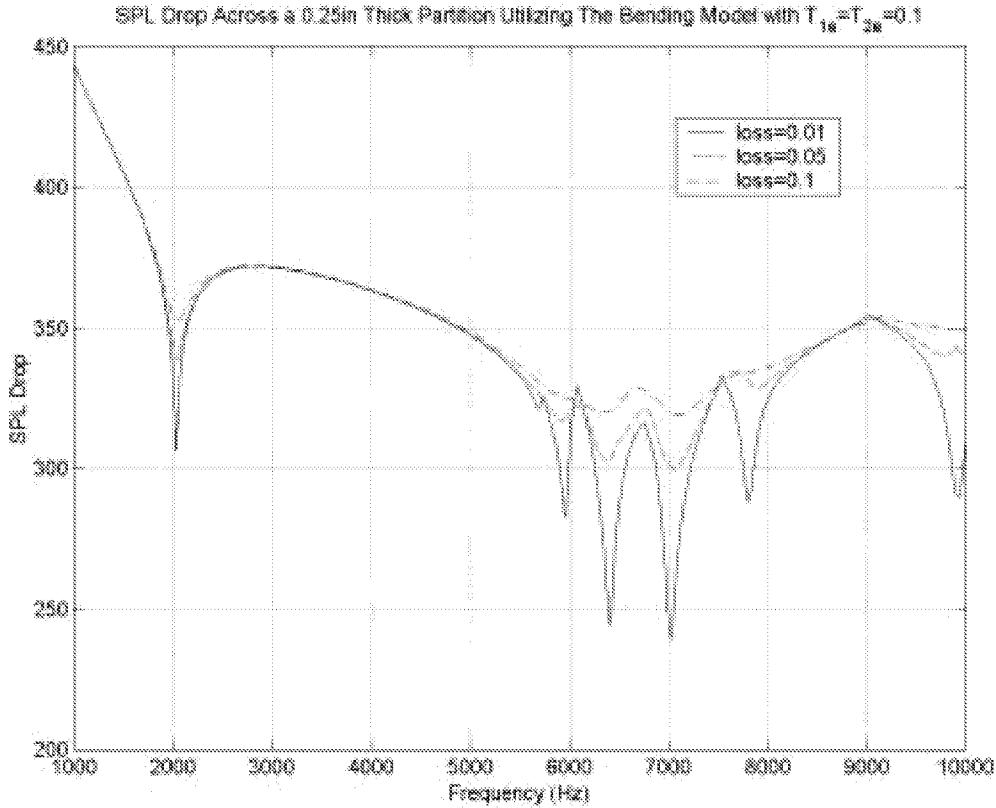
The results shown in Figs. 3-1 and 3-2 demonstrate that the partition would provide adequate isolation between the chambers of the DWNIT, if the plate partition was infinitely long and only compressive waves were present. The pseudo-plate model developed in Section 2.2.2 will be analyzed next to determine the effects of bending waves on the partition.



**Figure 3-3.** Intensity Transfer Function for a 0.25in (0.635cm) thick partition utilizing the Bending Model.

Figure 3-3 shows the intensity transfer function  $T_{12}$  for a 0.25in (0.635cm) thick steel partition utilizing the bending model. Figure 3-4 shows the corresponding SPL drop across a 0.25in (0.635cm) thick steel partition with  $T_{1e}=T_{2e}=0.1$ . These figures are essentially mirror images of each other, where high transmission produces low SPL drop and vice versa. These figures show low levels of the intensity transfer function and high SPL drops across the partition. Figure 3-4 shows that the minimum SPL drop across the

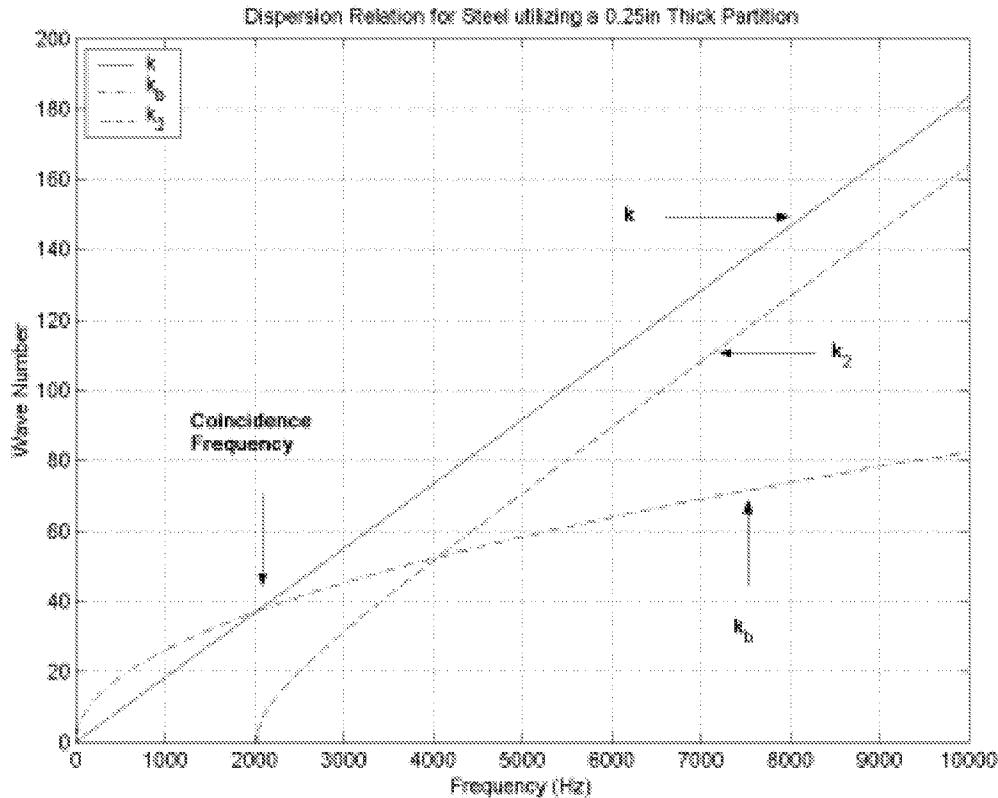
partition is around 245dB with a loss factor of 0.01. This SPL drop is well above the 40dB minimum design requirement.



**Figure 3-4.** SPL drop across a 0.25in (0.635cm) thick partition utilizing the Bending Model with  $T_{1e}=T_{2e}=0.1$ .

The first peak in the Fig. 3-3 and dip in Fig. 3-4 is the coincidence or critical frequency. This frequency occurs when the bending wave number and the incident wave number along the length of the plate match, as seen in Fig. 3-5. In essence at this point, the projected axial acoustic wavelength matches that of the plate, thereby making the plate transparent to the incident acoustic wave. Therefore, without damping at this frequency, all the incident acoustic waves would get transmitted. This would cause an increase in the intensity transmission and a reduction in SPL drop across the partition, as seen in Figs. 3-3 and 3-4. Since  $k_2$  is essentially imaginary below (for no damping) this critical frequency, there is no acoustic transmission across the plate until the coincidence

frequency is reached. From Figs. 3-3, 3-4, and 3-5, the coincidence frequency for the 0.25 in steel partition plate is around 2000Hz.

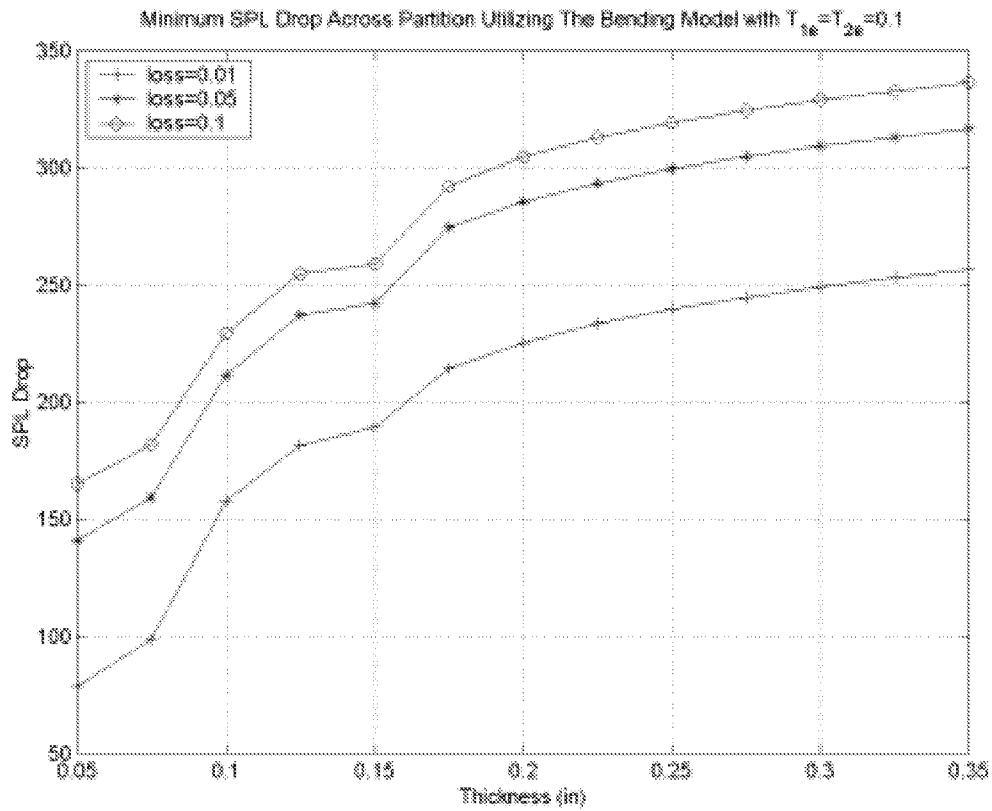


**Figure 3-5.** Dispersion relationship utilizing a 0.25in (0.635cm) thick partition.

The second set of peaks and dips in Figs. 3-3 and 3-4, respectively, are the partition's bending modes. The first bending mode occurs just under 6000Hz. The figure shows that a moderate increase in damping loss from 0.01 to 0.05 can significantly increase this minimum SPL drop from 245dB to 300dB. This increase in damping can be achieved by using commercially available viscoelastic damping polymers.

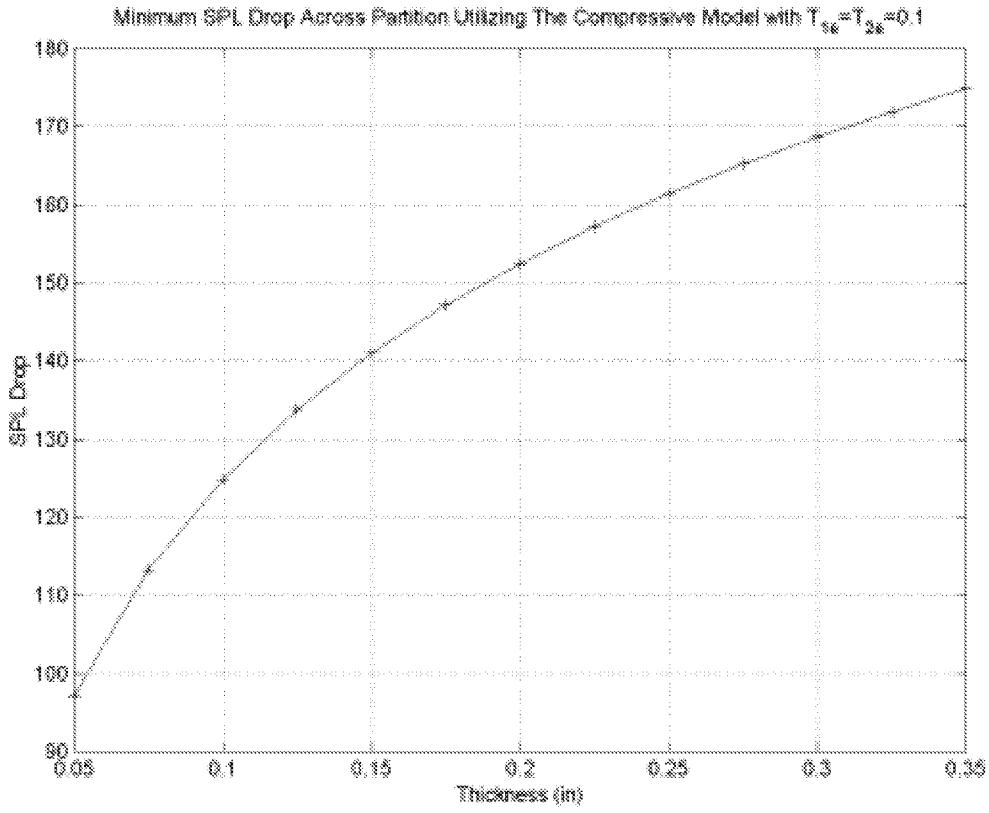
Figure 3-4 showed that 0.25in (0.635cm) thick steel partition would provide at least 40dB isolation between the two chambers of the DWNIT for a range of loss factors between 0.01 and 0.1. The next step in the design process was to determine how thin the partition could be made, and how much structural damping loss factor would be required to comply with the minimum allowable SPL drop design requirements.

Figure 3-6 shows the lowest SPL drop over the 1 to 10kHz frequency range, for plate thickness of 0.05in to 0.35in and loss factors between 0.01 and 0.1, utilizing the pseudo-plate model. Figure 3-7 shows the corresponding compressive model results. Figure 3-6 also shows that adding a small amount of damping greatly increases the performance of the partition, and that adding more than this level does not greatly improve the partition's performance.



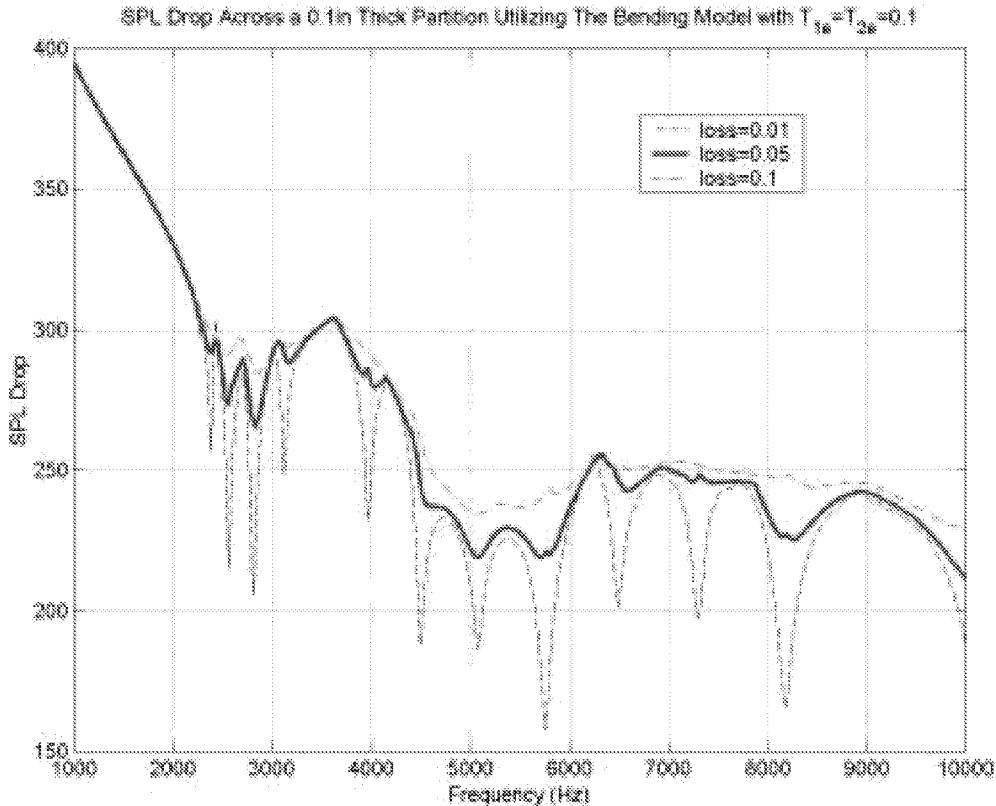
**Figure 3-6.** Minimum SPL Drop vs. Partition Thickness Utilizing the Bending Model.

Figures 3-6 and 3-7 show the graphical solution of the constrained minimization in Eq. (2-43) using the bending and compressive models, respectively. Both plots show that, for this range of thickness and damping values, all SPL drop values are above the 40dB design requirement. From the constrained minimization perspective, the thickness constraint in Eq.(2-43) is binding. Therefore, from the plot, the solution to Eq. (2-43) is  $\text{dB\_Drop}(\tau, \gamma) = 75\text{dB}$  (Bending),  $97\text{dB}$  (Compressive), where  $\tau = 0.05\text{in}$  and  $\gamma = 0.01$ .



**Figure 3-7.** Minimum SPL Drop vs. Partition Thickness Utilizing the Compressive Model.

Although the plate thickness and damping ranges in these figures satisfy the design requirements, simply selecting the solution to Eq. (2-43) might not be recommendable. Both Figs. 3-6 and 3-7 show that, as the partition thickness decreases, the SPL drops more rapidly across the plate. Therefore, small variations in thickness and damping due to manufacturing and installation process might put the partition design out of tolerance. Possible reasonable values of damping and thickness are 0.05 and 0.1in (0.127 and 0.254cm), respectively. This damping/thickness combination is high enough in SPL drop and shallow enough in terms of the slope of this curve to provide a “safe” design.



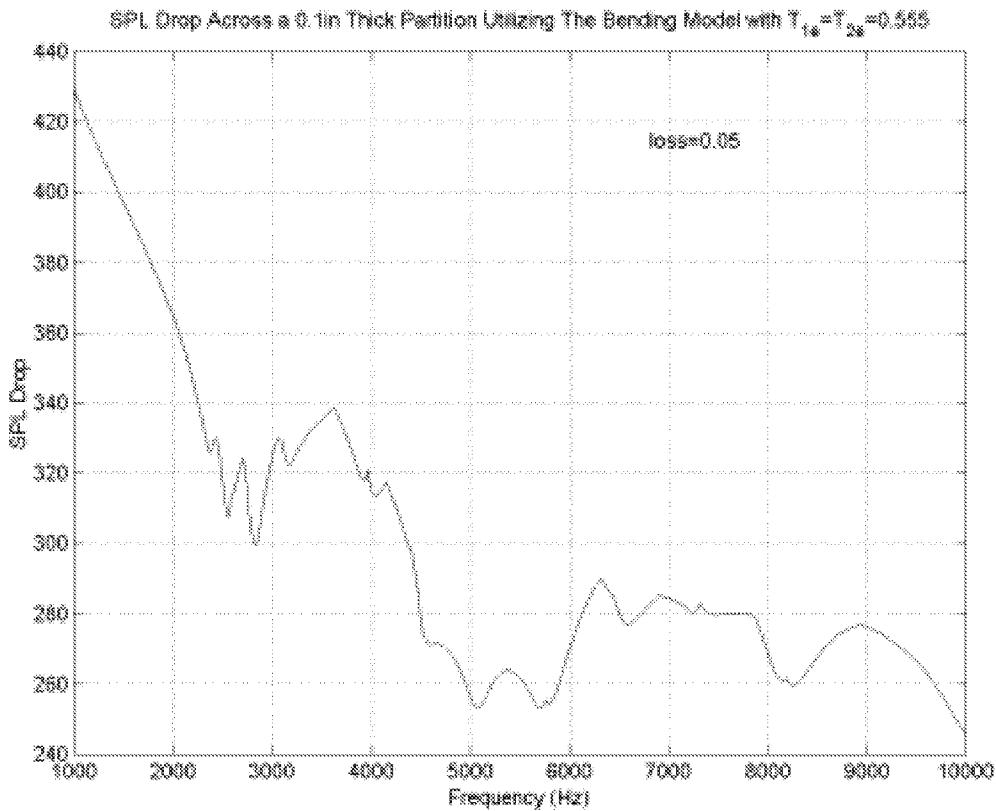
**Figure 3-8.** SPL drop across a 0.1in (0.254cm) thick partition utilizing the Bending Model with  $T_{1e}=T_{2e}=0.1$ .

Figure 3-8 shows the SPL drop across a 0.1in (0.254cm) steel plate partition utilizing the bending model. As expected, the plot shows that there are more bending modes present with a 0.1in (0.254cm) thick plate than with the original 0.25in (0.635cm) plate thickness design. Nevertheless, using a 0.05 loss factor damps out most of the mode dips. The lowest SPL drop under this design is about 225dB.

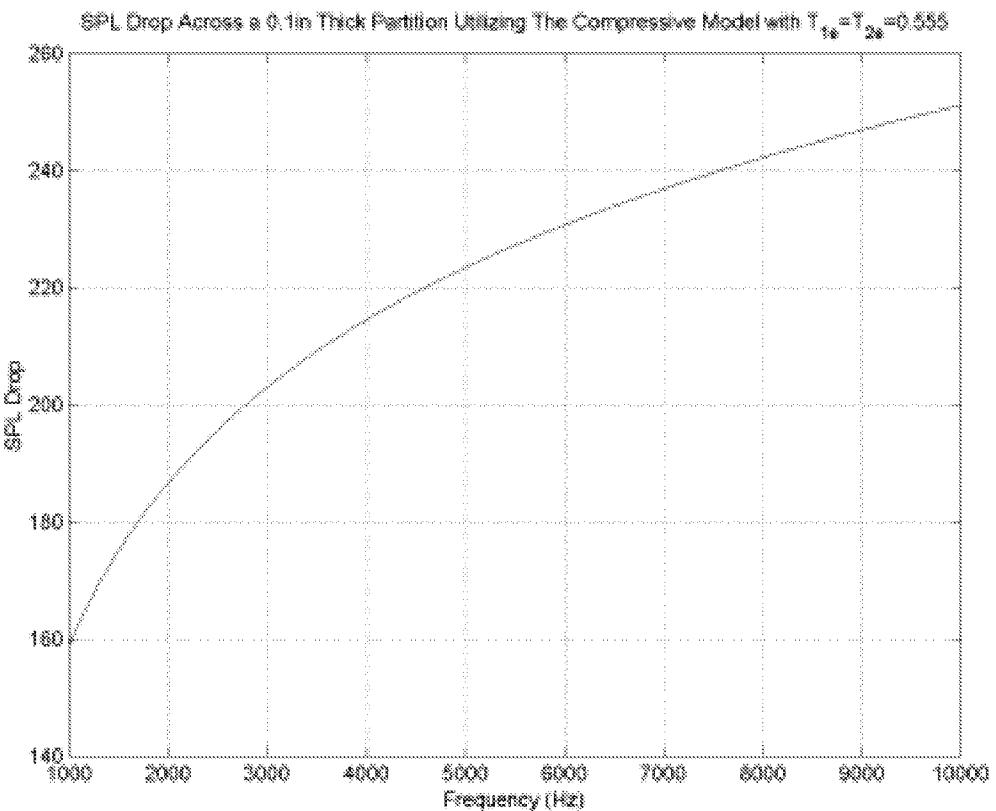
It is important to note that the design analysis so far has been conducted assuming that the end intensity transfer functions  $T_{1e}$  and  $T_{2e}$  are equal to 0.1. In other words, the end samples are reflective and their absorptivity is small. Although this is a good limiting assumption for the design, in reality normal incidence tubes are used to test very absorptive materials that would attenuate sound in noise control applications. For applications at NASA Langley's NIT, the end sample impedances rarely exceed 5pc. The intensity transfer function can be related to the impedance by

$$T = \frac{4\theta}{(\theta+1)^2 + \chi^2} \quad (3-1)$$

where  $\theta$  and  $\chi$  are the nondimensional resistance and reactance. Letting  $\theta$  equal 5 and  $\chi$  equal zero in Eq. (3-1) yields  $T=0.555$ . Figures 3-9 and 3-10 show the SPL drop across the 0.1in (0.254cm) partition with  $T_{1e}=T_{2e}=0.555$  using the bending and compressive model, respectively. From the figures, the minimum SPL drop across the partition is 250dB and 160dB for the bending and compressive models, respectively.



**Figure 3-9.** SPL drop across a 0.1in (0.254cm) thick partition utilizing the Bending Model with  $T_{1e}=T_{2e}=0.555$ .



**Figure 3-10.** SPL drop across a 0.1in (0.254cm) thick partition utilizing the Compressive Model with  $T_{1e}=T_{2e}=0.555$ .

## 4. Conclusions

An energy model was developed of the two chambers of the DWNIT. This energy model determined the SPL drop across the two chambers assuming an intensity transfer function for the chamber's partition. The model allowed the end sample intensity transfer functions to vary to determine their effect on the SPL drop across the partition. To model the partition, two intensity transfer function models were studied. The first intensity transfer function model used was a simple compressive model and the other was a pseudo-plate bending model.

The initial partition design (2" high, 16" long, 0.25" thick) was predicted to provide at least 160 dB SPL drop across the partition with the compressive model, and at least 240 dB SPL drop with the bending model. These results were produced when the structural damping loss in the bending model was 0.01 and the model end chamber sample transmissions were set to 0.1. This was done to analyze the partition under a close to worst-case scenario.

Since these results predicted more SPL drop than required, a plate thickness optimization algorithm was developed. The results of the algorithm routine indicated that a plate with the same height and length, but with a thickness of 0.1" and 0.05 structural damping loss, would provide an adequate SPL isolation between the chambers. This partition design would provide 210 dB and 125 dB SPL drop across the partition, utilizing the compressive and bending model, respectively. These results were also computed with the end chamber sample transmissions set to 0.1. When more realistic end intensity transmission coefficients (e.g., 0.555) were used, the compressive and bending model predicted a partition isolation of 250dB and 160dB, respectively.

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